Solve $\sinh x = 1$ by using the exponential definition of $\sinh x$ and an algebraic substitution $z = e^x$. **SCORE: ______ / 6 PTS**

$$\frac{e^x - e^{-x}}{2} = 1$$

$$\boxed{\frac{z - \frac{1}{z}}{2} = 1} \quad 1$$

$$z - \frac{1}{z} = 2$$

$$z^2 - 1 = 2z$$

$$\boxed{z^2 - 2z - 1 = 0} \quad 1$$

$$z = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} \quad \tfrac{1}{2}$$

EITHER IS OK

$$= \frac{2 \pm 2\sqrt{2}}{2} = \underline{1 \pm \sqrt{2}} \quad 1$$

$$\underline{z = e^x > 0} \quad 1$$

SO $z = 1 + \sqrt{2}$

$$e^x = 1 + \sqrt{2}$$

$$\underline{x = \ln(1 + \sqrt{2})} \quad 1\tfrac{1}{2}$$

Write and **prove** a formula for $\cosh(x-y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: ______ / 6 PTS**

$\cosh x \cosh y - \sinh x \sinh y$, 2

$$= \frac{e^x+e^{-x}}{2} \cdot \frac{e^y+e^{-y}}{2} - \frac{e^x-e^{-x}}{2} \cdot \frac{e^y-e^{-y}}{2}$$ 1

$$= \frac{\cancel{e^{x+y}} + e^{x-y} + e^{y-x} + \cancel{e^{-x-y}} - (\cancel{e^{x+y}} - e^{x-y} - e^{y-x} + \cancel{e^{-x-y}})}{4}$$ 2

$$= \frac{2e^{x-y} + 2e^{y-x}}{4}$$

$$= \frac{e^{x-y} + e^{-(x-y)}}{2}$$ 1

$$= \cosh(x-y)$$

Prove that $g(x)=\ln(x+\sqrt{x^2+1})$ is the inverse of $f(x)=\sinh x$ by simplifying $g(f(x))$. **SCORE: ______ / 5 PTS**

$$\ln(\sinh x+\sqrt{\sinh^2 x+1})$$ 1

$$=\ln(\sinh x+\sqrt{\cosh^2 x})$$

$$=\ln(\sinh x+\cosh x)$$ 1

$$=\ln\left(\frac{e^x-e^{-x}}{2}+\frac{e^x+e^{-x}}{2}\right)$$ 1

$$=\ln\frac{\cancel{2}e^x}{\cancel{2}}$$

$$=\ln e^x$$ 1

$$=x$$ 1

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry. **SCORE: ______ / 4 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **NOTE: You do NOT need to prove the identity.**

$$\underbrace{\cosh^2 x - \sinh^2 x = 1}$$ 1

[b] Divide both sides of that identity by $\sinh^2 x$ and simplify.

$$\underbrace{\coth^2 x - 1 = \operatorname{csch}^2 x}$$ 1

[c] If $\coth x = 3$, find $\sinh x$.

$\frac{1}{2}$

EITHER IS OK

$$3^2 - 1 = \operatorname{csch}^2 x$$

$$\operatorname{csch}^2 x = 8$$

$\frac{1}{2}$ $\underbrace{\operatorname{csch} x = \pm 2\sqrt{2}}$

$$\coth x = \frac{\cosh x}{\sinh x}$$

SINCE $\underbrace{\coth x \text{ AND } \cosh x > 0}$, $\frac{1}{2}$

THEREFORE $\sinh x > 0$

SO $\operatorname{csch} x = 2\sqrt{2}$

$\frac{1}{2}$ $\underbrace{\sinh x = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}}$ $\frac{1}{2}$

Rewrite $\operatorname{sech}(3\ln 2)$ in terms of exponential functions and simplify. **SCORE: _____ / 3 PTS**

$$\boxed{\frac{2}{e^{3\ln 2}+e^{-3\ln 2}}} = \frac{2}{e^{\ln 8}+e^{\ln \frac{1}{8}}} = \boxed{\frac{2}{8+\frac{1}{8}}} = \boxed{\frac{16}{65}}$$

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